CAPNOGRAPHY AND THE BAIN CIRCUIT I: A COMPUTER MODEL

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ABSTRACT. The Mapleson D anesthesia breathing system has no valves and allows rebreathing of carbon dioxide. Its coaxial version is known as the Bain system. The interpretation of capnograms obtained during its use requires an understanding of the interrelationships of patient and system variables. Toward that end, a systematic description of mechanical ventilation with the Bain circuit was undertaken based on the physical laws of gas transport. The mathematical formulation of the model contains the relations between pressure, flow, and volume in the tube, alveolar space, and ventilator. The flows, calculated from these relations, are used to determine the CO_2 concentrations in the different parts of the model. Two sets of data are used—patient and system. The patient data, used to solve the equations numerically, are lung-thorax compliance, CO_2 inflow into alveolar space (CO_2 production), functional residual capacity, dead space volume, airway resistance, and respiratory quotient. The ventilation system data comprise the dimensions and volumes of the Bain circuit, ventilator, connectors, and tubes; spill valve pressure; resistances to flow in the individual tube parts; ventilator settings; and fresh-gas flow rates.

After incorporation of a volunteer's respiratory variables into the model, capnograms obtained from the model compared well with those obtained from the volunteer.

The structure of the model is such that it permits easy introduction or changes of patient and system variables to obtain individual results or model specific circumstances. This flexibility makes it a useful tool for understanding the properties of the Bain circuit under a variety of clinical circumstances. The results may be displayed in a number of different ways.

KEY WORDS. Equipment: Bain circuit; Capnography; Computer model; Monitoring: Carbon dioxide; Mapleson system.

The Bain modification of the Mapleson D system (the Bain circuit) was originally described by Bain and Spoerel [1] as a lightweight, compact anesthetic system. It differs from circle systems in that it permits rebreathing of expired CO_2. It is designed to operate without valves. When the fresh gas flow falls below peak inspiratory flow rate, part of the system becomes dead space, i.e., exhaled gases containing CO_2 are reinspired. To maintain a desirable arterial carbon dioxide tension ($PaCO_2$), the anesthetist can vary fresh gas flow and/or minute ventilation, as well as respiratory rate and inspiratory-to-expiratory (I:E) time ratios. However, pulmonary function variables, CO_2 production, and the respiratory quotient also influence the overall performance of the circuit.

The fresh-gas flow rate needed to avoid rebreathing, is related to $\dot{V}_E$ [2,3]. Other studies relate the required fresh gas flow to body surface area [4] or to body weight

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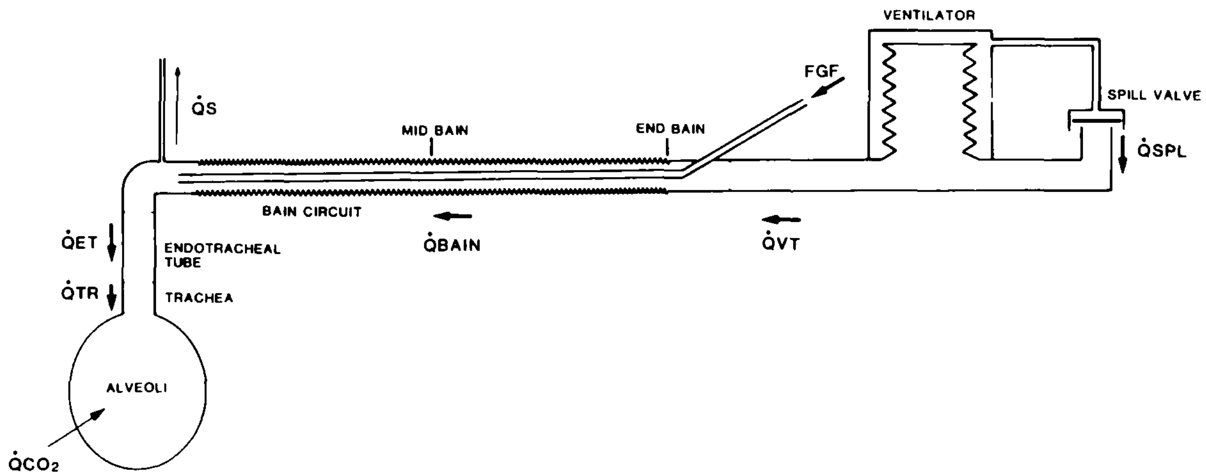

Fig 1. Schematic representation of the Bain circuit as used in the model. From left to right: alveoli representing the lungs of the individual being simulated; trachea; endotracheal tube; elbow connector; actual Bain circuit with inner tube for fresh gas supply; connecting tube to the ventilator; and ventilator. FGF = fresh gas flow; $\dot{Q}CO_2$ = CO_2 production; $\dot{Q}TR$, $\dot{Q}ET$, $\dot{Q}S$, $\dot{Q}BAIN$, $\dot{Q}VT$, and $\dot{Q}SPL$ = flow rates in the trachea, endotracheal tube, sampling tube, Bain circuit, ventilator, and spill valve, respectively. Arrows indicate positive directions of flow.

[5,6,7]. There is a wide range of recommended fresh-gas flow rates for use with the Bain circuit, from 70 ml/kg/min [2,3] to 300 ml/kg/min [8].

Gabrielsen et al [9] found that the extent of rebreathing also depended on the breathing pattern, particularly on the length of the ventilatory pause. Seeley et al [10] undertook a theoretical and experimental study of the Bain circuit. On the basis of animal studies, they designed a nomogram that predicted $PaCO_2$ from the minute ventilation and fresh-gas flow rate. In their analysis, they assumed a uniform mixing of gases in the tube that constitutes the Bain circuit.

The complex interrelations between all the factors involved can be resolved only by a detailed analysis of the system. We therefore designed a systematic description of the Bain system and its interaction with the ventilator and the patient. This description is based on the physical laws of gas transport and employs the multiple model approach developed by Benecken and Rideout [11].

This paper describes the approach to such a mathematical description, or model, the basic equations involved, and their solution for the case of mechanical ventilation with the Bain circuit. The results are shown and compared with measurements obtained from a volunteer whose lungs were mechanically ventilated. Future extension of the model will permit the study of spontaneous and manual ventilation.

METHODS

The patient's endotracheal tube, the Bain circuit, and the connection of the Bain circuit with the ventilator can be regarded as a straight tube with different diameters for the individual components. Figure 1 shows a schematic of this arrangement.

We calculate how CO_2 is stored and transported as a result of gas moving back and forth under the influence of the ventilator and the fresh-gas flow rate. We regard the total amount of gas, which may consist of O_2, CO_2, N_2O, and other anesthetic gases, as the carrier gas; the concentration, or the partial pressure of carbon dioxide (Pco_2), is considered the carried substance.

The description of CO_2 transport is basically a two-dimensional problem, that is, distance along the tube is the first dimension and time is the second. Therefore the mathematical equations take the form of partial differential equations.* Since the driving functions (inputs, such as breathing pattern) are in general nonanalytical functions† and the equations are solved by digital computers, the length and time dimensions are both divided into small, discrete units. Thus, the tube is divided into a number of segments.

The basic assumption is that within each tube segment there is a uniform distribution of CO_2 molecules. The method for calculating the concentration of particles is shown in Figure 2, with t_1, t_2, and t_3 representing three consecutive instants of time. During t_1 and t_2, the volume flow of the carrier moves a certain fraction of

* A partial differential equation contains at least one partial derivative. A partial derivative has several variables; only one varies, while the others remain constant, i.e., are parameters.

† A nonanalytical function does not have a derivative at each point, i.e., it is a discontinuous function.

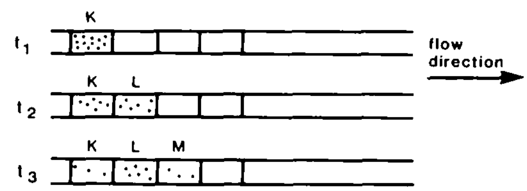

Fig 2. A tube divided into segments and represented at three consecutive times, t_1, t_2, and t_3, illustrates how several particles are transported to neighboring segments under the influence of a carrier flow from left to right.

the particles from segment K to segment L At t_2, the number of particles in K is reduced because of the inflow of the carrier without particles. Based on this new distribution and the known carrier volume flow, the distribution at t_3 can be calculated. This calculation process is repeated continually and yields not only the time course of the particle concentration in a particular segment by looking at a segment, for example, K, as a function of time, but also the longitudinal distribution of particles along the tube at a particular instant.

When the direction of the flow is reversed, particles will no longer enter segment L from K but from M instead. This change in calculating the number of particles in segment L results from the change in the flow direction of the carrier. Thus, there is a distinct cause-and-effect relation: the carrier flow is the cause and the particle distribution is the result.

At branching points of the anesthesia circuit, where gas is added (e.g., fresh gas) or removed (e.g., sampling flow for a gas analyzer), the same general approach is used: (1) The carrier flows are established in the different branches, according to the law of mass conservation and (2) for each subsequent instant of time, the new distribution of particles is calculated on the basis of both the previous distribution in all branches and the known carrier flows.

Simplifications

Each model of a physical system brings with it a number of assumptions, usually in the form of simplifications. The following simplifications were made:

The patient's lungs are represented by one segment with a uniform CO_2 distribution; the airways by their volume and their resistance to flow.
The ventilator volume is represented by one segment.
The velocity profile is assumed to be flat (plug flow) because the corrugations in the tube are assumed to cause small eddies, which, in turn, result in good cross-sectional mixing.

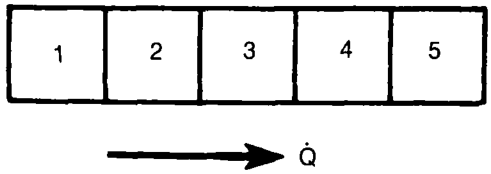

Fig 3. Calculation of CO_2 transport is illustrated by numbered blocks representing five segments of a tube. $\dot{Q}$ = flow.

Axial diffusion is taken into account by adjusting the time intervals and the segment-size (see Discussion).
Within each segment, complete mixing takes place.
Gas is assumed to be incompressible over the encountered pressure range.
Compliance is neglected over the same pressure range, except in small children, whose lung-thorax compliance is approximately that of the system compliance.
Temperature is assumed to be constant.
Uptake and elimination of anesthetic gases are not considered, since these are transient phenomena.

Mathematical Formulation of the Tube Model

The general principle is illustrated by a tube subdivided into five segments (Fig 3). We assume that each segment has a volume equal to V. This assumption is for simplicity, and no loss of generality is introduced. The carrier gas volume flow through the tube from left to right is equal to $\dot{Q}$. In addition, the following abbreviations are used in the description:

T	the moment in time, or time instant, at which the distribution is calculated
DT	the time interval, or difference in time, between two consecutive time instants
$F_1 \ldots F_5$	the fraction of CO_2 in each of the five segments
S	the shift factor (a dimensionless group introduced for mathematical simplicity), later defined as $\dot{Q} \times DT/V$

Equation 1 calculates the CO_2 volume in segment 3 at instant T.

$$V \times F_3(T) = V \times F_3(T - DT) + \dot{Q}(T - DT) \times DT \times F_2(T - DT) - \dot{Q}(T - DT) \times DT \times F_3(T - DT) \quad (1)$$

The term to the left of the equal sign represents the volume of CO_2 in segment 3. The first term to the right of the equal sign is the CO_2 volume at time instant $(T - DT)$; the second term is the amount of CO_2 that entered

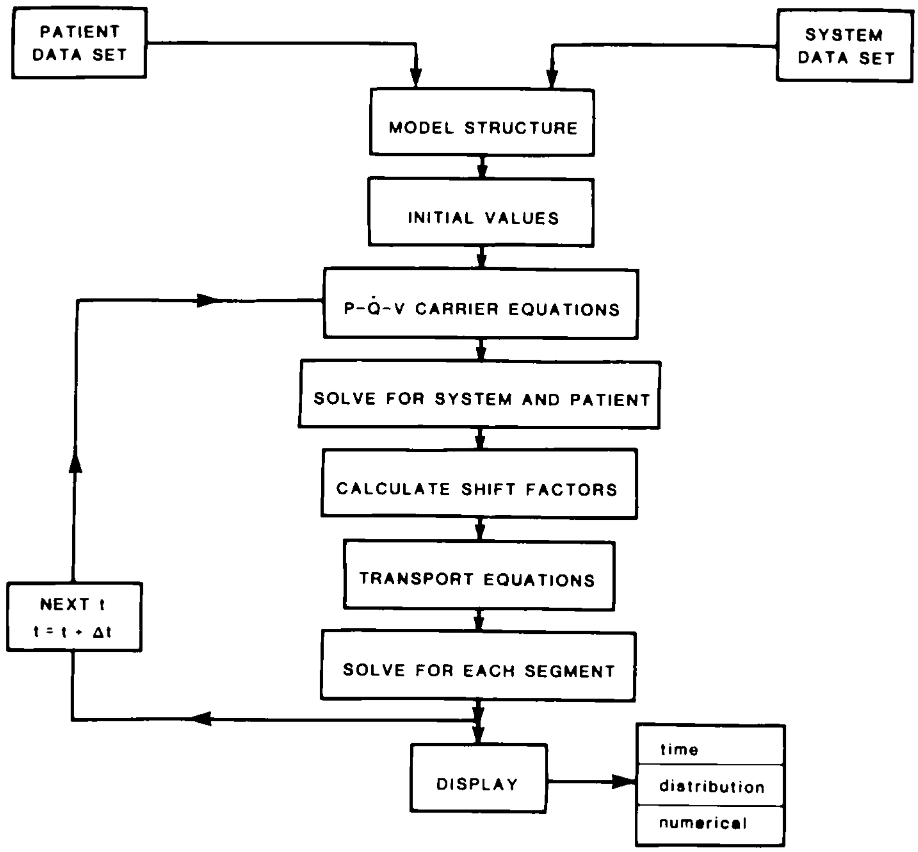

Fig 4. General structure of the computer program. t = *time instant measured;* P = *pressure;* Q̇ = *flow;* V = *volume.*

segment 3 from segment 2 during the time interval DT and the last term is the amount of CO_2 that leaves segment 3 during the time interval DT. All values to the right of the equal sign are taken at time instant $(T - DT)$, and therefore lead to the new F_3 value at time instant (T).

Dividing by V and introducing the shift factor, $S = \dot{Q}(T - DT) \times DT/V$ gives equation 2.

$$F_3(T) = F_3(T - DT) + S \times [F_2(T - DT) - F_3(T - DT)] \quad (2)$$

The shift factor, S, thus becomes dependent on the carrier flow rate, $\dot{Q}$, the time interval, DT, and the segment volume, V. If the direction of the flow reverses, CO_2 is transported from segment 4 into segment 3 and from segment 3 into segment 2. In that case, the CO_2 fraction in segment 3 is calculated using equation 3.

$$F_3(T) = F_3(T - DT) + S \times [F_4(T - DT) - F_3(T - DT)] \quad (3)$$

It should be realized that S, being dependent on the carrier flow rate, is different in equations 2 and 3, and that $F_2(T - DT)$ in equation 2 is replaced by $F_4(T - DT)$ in equation 3.

Similar equations describe the CO_2 fractions in all segments considered. The simultaneous solution for one instant, T, of all equations involved is then repeated to generate the time course of CO_2 distribution during one or more respiratory cycles. This calculation also yields quantitative information about the longitudinal distribution of CO_2 by displaying, at one instant, the different fractions F_1 through F_5.

In the Appendix we present and discuss not only the equations that describe the pressure-flow-volume relations in the entire patient-ventilator system but also the calculation of the CO_2 fractions in the alveolar space and the ventilator.

General Structure of the Model

To solve the entire set of equations repeatedly for each time interval, the set has been programmed using FORTRAN. The program runs on either a Digital Equipment Corporation LSI 11/23 or an IBM-PC/XT computer.*

Figure 4 outlines the general structure of the model. The data necessary to initiate the model consist of two sets: a patient data set and a system data set. Numerical data have to be given, since the model described here operates with numbers. If no data about a particular patient are available, data for an "average" patient can be introduced. The system data set should contain the data about the particular ventilator arrangement. The patient data set comprises:

Lung thorax compliance
CO_2 inflow into the alveolar space (or CO_2 production)
Respiratory quotient, accounting for inequalities of inspired versus expired volumes
Functional residual capacity
Dead space volume
Airway resistance

The system data set comprises:

Dimensions and volumes of ventilator, tube and connectors
Spill valve pressure setting
Resistances to flow in the different segments of the tubes
Ventilator settings, such as I:E ratio, respiratory rate, and minute ventilation
Fresh-gas flow rates

On the basis of these two data sets, the model structure can be defined and the number of segments for each tube section selected. A value for the time interval, *DT*, should also be selected at this stage.

From the definition of the shift factor, *S*, given above, it will be clear that *S* may never exceed a value of 1. If it did, the product, $\dot{Q} \times DT$, would become greater than the actual segment volume, *V*, and gas would be transported beyond the limit of the segment during the time interval, *DT*, an occurrence that is not allowed in this type of step-by-step calculation of the CO_2 fractions. Therefore, a limit is set on the choices of segment size, *V*, and the time interval *DT*.

Volunteer and System

Table 1 gives the combined volunteer and system data as they were used to initiate the model. The functional residual capacity of the volunteer (weight, 80 kg; height, 178 cm), who passively allowed himself to be ventilated, was measured in the pulmonary function laboratory. Tidal volume was measured using a pneumotachograph; the remaining values were estimated.

The data for the ventilator (Ohio 7000 Electronic Ventilator) and the tubes were obtained from the factory specifications or measured. Tube volumes were measured by water displacement and their resistance to flow from simultaneous measurements of flow rates through and pressure differences across the tubes. The quadratic pressure-flow characteristics of the spill valve and the exhaust valve (which controls the outflow rate during filling of the ventilator bellows) were estimated on the basis of their construction and the measured pressure and flow curves.

The ventilator settings and the fresh-gas flow rate were adjusted for comfortable ventilation of the volunteer. To illustrate the model, we selected two fresh-gas flow rates, 10.2 and 12.2 L/min, for comparing the

Table 1. Numerical Values for Volunteer and System Variables

Variable	Value
VOLUNTEER	
Lung-thorax compliance	133 ml/mm Hg
CO_2 production	210 ml/min
Respiratory quotient	1.0
Functional residual capacity	3,900 ml
Dead space volume	105 ml
Tidal volume	660 ml
Airway resistance	0.0012 mm Hg/ml/min
SYSTEM	
Volume of Bain outer tube	510 ml
Volume of elbow connector	15 ml
Volume of connecting tube between Bain circuit and ventilator	530 ml
Maximum volume of ventilator	2,300 ml
Resistance of Bain and connecting tube	0.003 mm Hg/ml/min
Spill valve pressure setting	2 mm Hg
Sampling flow to capnograph	205 ml/min
Quadratic pressure-flow relation through spill and exhaust valves	$\dot{Q} = 12 \times \sqrt{P}$ ($\dot{Q}$ in L/min; P in mm Hg)
Fresh gas flow	10.2 and 12.2 L/min
VENTILATOR	
I:E time ratio	1:2
Respiratory rate	7.75 breaths/min
Minute volume	5 L/min

$\dot{Q}$ = carrier gas volume flow; P = pressure; I:E = inspiratory to expiratory.

*A copy of the program can be obtained from J.S.G. The display subroutines may require adjustments to meet local hardware arrangements.

Table 2. Number and Volume of Segments for Each Part of System (Two Models)

	No. of Segments		Volume per Segment(ml)	
Portion of Bain Circuit	16-Segment Model	36-Segment Model	16-Segment Model	36-Segment Model
Alveoli	1	1	3,900	3,900
Trachea and endotracheal tube	3	7	35	15
Elbow connector	1	1	15	15
Bain tube	5	13	102	39.2
Ventilator hose	5	13	106	40.7
Ventilator	1	1	2,300	2,300

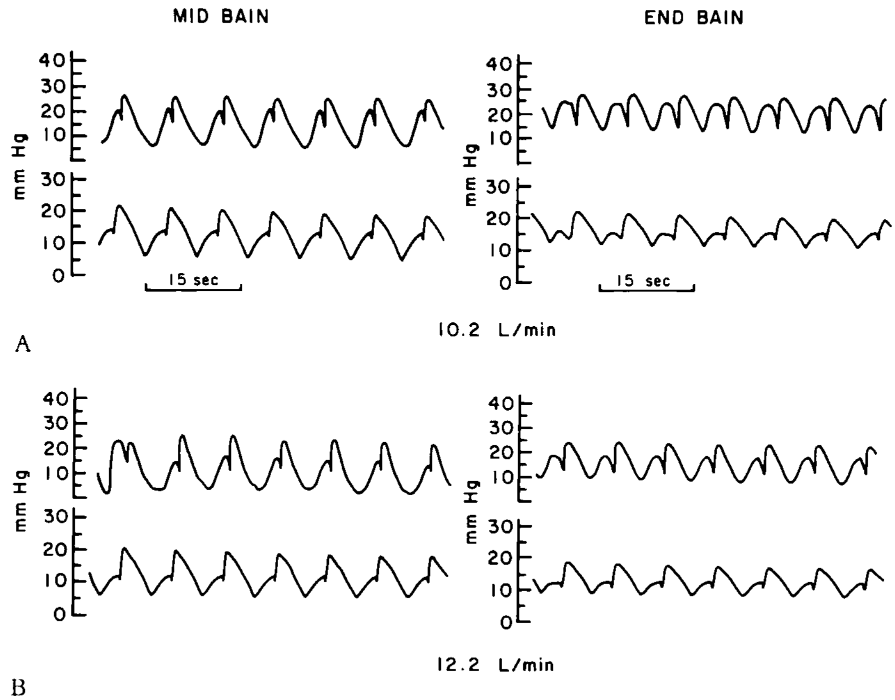

Fig 5. A comparison of capnograms from a conscious volunteer and from results of the computer model, using the same respiratory variables and two different fresh-gas flow rates, 10.2 L/min (A) and 12.2 L/min (B). In each of the four pairs of traces, the upper trace is from the volunteer, the lower from the computer model. The volunteer allowed the ventilator to assume control of ventilation. Capnograms were obtained by collecting and analyzing gas from the expiratory (outer) tube of the Bain circuit in the middle and at its distal end (see Figure 1). Time progression is from left to right.

volunteer's capnograms with those generated by the model.

Table 2 lists the number of segments used to represent different components of the patient-ventilator system and the resulting volumes per segment, for two different models (consisting of 16 and 36 segments, respectively) of the same "volunteer-ventilator" arrangement.

The pressure-flow-volume equations (see Figure 4) are solved yielding the set of shift factors subsequently necessary to solve the transport equations. The computer calculates the variables at the next time instant, $T + DT$, based on the results at time T and the data sets. The data can be stored and displayed in various ways. The data can be stored and displayed in various ways. The calculations can continue for any number of respiratory cycles, depending on the particular study.

RESULTS

Capnograms obtained from the male volunteer, ventilated according to the data given in Table 1, are compared with the results obtained from the 16-segment computer model by using the same data (Fig. 5).

Figure 6 compares capnograms generated at two

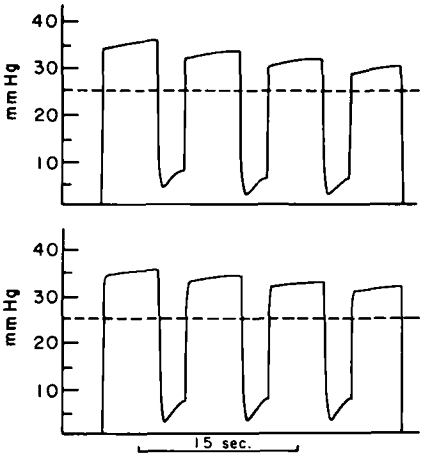

Fig 6. Computer-generated capnograms obtained from distal endotracheal segment during the first four breaths after onset of the computation, using the data in Table 1. At the onset of the first simulated breath, the endotracheal tube and the alveoli are assumed to be filled with gas containing CO_2 at a partial pressure of 40 mm Hg. The mouthpiece in which the sampling took place was assumed to be free of CO_2. Fresh-gas flow rates are 12.2 L/min (top) and 10.2 L/min (bottom). Time progression is from left to right.

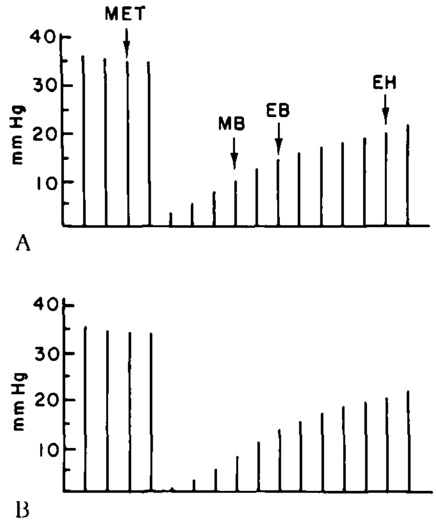

Fig 7. Longitudinal presentation of partial pressures of CO_2 (PCO_2) in the ventilation circuit at end expiration, calculated by the 16-segment computer model at fresh-gas flow rates of 10.2 L/min (A) and 12.2 L/min (B), using the data in Table 1. The height of each vertical bar represents the PCO_2 in a specific segment; multiplication by the respective segmented volume yields the actual CO_2 volume stored in each of the segments. The first bar on the left represents PCO_2 in the alveoli; the next three bars represent PCO_2 in three segments of the endotracheal tube; thus, the height of the third bar from the left (the midendotracheal tube segment [MET]) equals the PCO_2 at end expiration in the capnograms in Figure 6. The shortest bar in the graph represents PCO_2 in the right-angle elbow connector, the next two bars that in the Bain tube and in the hose that connects the Bain circuit with the ventilator, respectively. The bar farthest to the right depicts PCO_2 as measured from the ventilator. MB = middle of Bain tube, EB = end of Bain tube, EH = end of ventilator hose closest to the ventilator. See Figure 1 for locations of segments. The display is updated regularly and shows the back-and-forth movement of CO_2 in the Bain circuit.

fresh-gas flow rates by the 16-segment model. The curves reflect a washout to a lower alveolar Pco_2. The general appearance of these curves agrees with the results shown by Gabrielsen et al [9].

Figure 7 represents a longitudinal CO_2 distribution at end expiration after the second breath. It displays the CO_2 values in the 16-segment model at fresh-gas flow rates of 10.2 and 12.2 L/min. The lowest Pco_2 value is found in the segment that represents the right-angle elbow connecting the endotracheal tube to the Bain tube. This we call the "mixing chamber," since fresh gas flow and expired air meet here.

Table 3 shows the numerical output of the computer model that generated the capnograms and from which other plots could be constructed. Printouts provide selected values from a matrix of available data on pressures, flows, and partial pressures in any of the 16 segments. The printout shown in Table 3 depicts the transition from the first expiration to the second inspiration. It therefore represents the incomplete mixing of expired and fresh gases in the ventilator bellows, as is observed in such systems for the first few breaths. Any combination of variables needed for a particular study could be selected for numerical output.

The series of capnograms shown in Figure 8 allow an anaysis of CO_2 fluctuations in different parts of the system and a comparison of capnograms generated in a 16-segment versus a 36-segment model on the basis of the data given in Table 2.

DISCUSSION

Although many investigations and reports address the Bain system and strategies for assuring adequate removal of CO_2, we are unaware of a systematic description of this system. The valveless breathing circuit makes rebreathing very probable, yet difficult to predict. The measurement of CO_2 close to the endotracheal tube is difficult because of the possibility of fresh gas diluting any CO_2 present. Extra care must therefore be taken in sampling gases for analysis, as shown by Gravenstein et al [12].

Once a representative sample has been collected and analyzed, interpretation presents additional challenges, since the amount of CO_2 washed into the expiratory limb of the system, and hence to the ventilator and through the spill valve to the outside, is influenced not

Table 3. Values for Respiratory Flow and Pressure Using 16-Segment Model[a]

K	$\dot{Q}ET$ (ml/s)	PMIX (cm H_2O)	PCAL (mm Hg)	PCET (mm Hg)	PCMX (mm Hg)	PCBN (mm Hg)	PCHO (mm Hg)	PCVT (mm Hg)
740	−0.4	1.6	38.7	37.0	0.4	27.5	30.5	29.8
760	−0.2	1.6	38.9	37.0	0.2	27.1	30.4	29.8
780	−0.1	1.6	39.0	37.0	0.1	26.7	30.4	29.9
800	−0.0	1.6	39.2	37.0	0.0	26.2	30.3	30.0
20	333.4	2.7	39.2	12.9	5.5	26.9	17.5	2.9
40	333.4	3.2	39.2	8.4	7.9	27.5	10.4	2.8
60	333.4	3.7	38.9	9.0	10.1	27.8	6.7	2.7

[a]Numerical output from the model on the basis of the data listed in Table 1, with a fresh gas flow of 12.2 L/min. The period of measurement is the transition from the first to the second breath after the start of the computations.

K = number of 0.01-second time intervals; $\dot{Q}ET$ = flow in the endotracheal tube; PMIX = pressure in the elbow segment of the model; remaining values are partial CO_2 pressures in the alveolar segment (PCAL), the midendotracheal segment (PCET), the elbow segment (PCMX), the most distal Bain segment (PCBN), the most distal ventilator hose segment (PCHO), and the ventilator (PCVT).

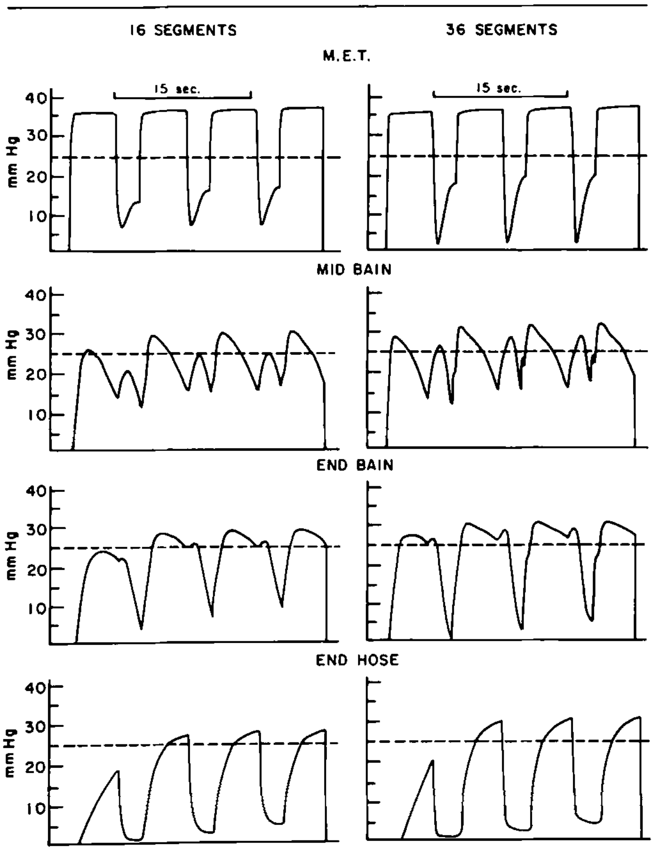

Fig 8. Computer-generated capnograms from different parts of the Bain circuit derived from the 16- and 36-segment models, using the data in Table 1 with a fresh-gas flow rate of 5.2 L/min. The four positions are indicated in Figures 1 and 7. M.E.T. = *midendotracheal tube.*

only by the amount of CO_2 exhaled by the patient, the fresh-gas flow rate, and the volume of the breathing tube, but also by the time course of expiration and the duration of the expiratory pause. This complex interaction has been taken into account in the computer model presented in this article.

We have discussed above the limits in selecting volumes of the segments and size of the time intervals because of the shift factor. If we assume, for illustration, that only one segment represents the entire tube, expired CO_2 would reach the ventilator immediately as a result of the assumed complete mixing. At the other extreme, an infinite number of segments would, under the same assumptions, simulate a distortion-free transport of any CO_2 distribution, because no axial diffusion can take place. Thus, the selection of the optimal number of segments is not simple. Comparison of the curves in Figure 8 shows differences between the results obtained from the 16-segment model and those obtained from the 36-segment model. The 36-segment model shows some finer details. The decision as to how many segments to use in the model depends entirely on the specific problem to be solved. For example, for studying long-term behavior and average rebreathed volumes of CO_2, 16 segments will be more than adequate. For the detailed interpretation of an independently recorded capnogram curve, 36 or even more segments may be needed. Because of the program structure, only three numbers have to be changed in the system data set (the number of segments for the endotracheal tube, Bain circuit, and connecting hose, respectively) to obtain a different degree of detail in the model.

Selection of the size of the time intervals is limited by still another factor, the accuracy of the calculations. Many of the equations involved are actually integrations, which may result in computation errors when the time interval chosen is too large. Time interval values in the range of 10 to 50 ms have resulted in no observable errors.

A comparison of the capnograms obtained from the volunteer and the model (see Figure 5) shows reasonable agreement. The general pattern is the same and the changes that result from an increase in the fresh-gas flow rate are in good agreement. Each cycle shows two distinct peaks, the lowest being related to inspiration and the highest to expiration. These peaks can be explained as follows: Peak expiratory flow rates are high, about 40 to 60 L/min, which results in minimal dilution by the fresh gas flow. Thus, a high flow rate of CO_2 initially will pass by the sampling site. In the later stage of expiration CO_2 is exhaled at smaller flow rates, therefore, the same fresh-gas flow rate produces a larger dilution effect. Figure 7 illustrates CO_2 distribution in the entire system at end expiration. With the next inspiration, only part of the gas stored in the tube will be rebreathed, as can be calculated from Table 1. Thus, gas with lower CO_2 concentrations will pass the sampling site during inspiration. Further comparison of the curves in Figure 5 shows that the increase in fresh gas flow lowered the overall partial pressure in the same way in both the volunteer and the model. In addition, the first peak was lower relative to the second as a result of the increase in fresh gas flow.

The computer model of the Bain system provides an excellent teaching tool by offering vivid evidence of CO_2 distribution in the expiratory limb of the Bain circuit, the connecting tube to the ventilator, and the ventilator bellows itself, as shown in Figure 7. The opportunity to observe the CO_2 distribution moving back and forth during the respiratory cycle as the computation evolves is most instructive. A similar understanding of these phenomena is hard to obtain with other methods of study, even from simultaneous recordings such as those shown in Figure 8.

This model could increase understanding of a variety of clinical problems. A small computer makes application of the model easily accessible, both for consultation and for tutorial purposes. Different combinations of tubes, valves, and other components can be tested for clinical efficacy. Further studies of such factors as the effect of leaks, increased resistances, or changed lung-thorax compliances on the amount of rebreathed CO_2 can be performed with this model.

APPENDIX

Pressure, Flow, and Volume Calculations

The calculation of volume, flow, and pressure data deals solely with the carrier gas behavior and is independent of the substance (CO_2) that is carried, since it is expressed in terms of fractions; the actual CO_2 component is also considered part of the carrier as exemplified in equation 8 below. During mechanical ventilation, inspiration is accomplished through a ventilator that generates the flow of gas to the patient. Expiration is generally passive; the pressure built up in the lungs is the driving force. Figure 1 shows the ventilator arrangement and the spill valve, which operates on a difference of pressure between the inside and outside of the bellows; also shown are the notations and positive directions of the various flows. Volume flow rate is denoted by $\dot{Q}$.

The following abbreviations are used in the descriptions of pressure, flow, and volume calculations:

$\dot{Q}BAIN$	flow rate in the Bain circuit
$\dot{Q}VT$	flow rate from or to the ventilator
VVT	volume of the ventilator
T	the moment in time, or time instant, at which the distribution is calculated
DT	the time interval, or difference in time between two consecutive time instants
$\dot{Q}SPL$	flow rate through the spill valve
$\dot{Q}ET$	flow rate in the endotracheal tube
FGF	fresh gas flow
$\dot{Q}S$	sampling flow rate
$\dot{Q}TR$	flow rate in the trachea
VAL	volume of the alveoli
RQ	respiratory quotient
$\dot{Q}CO_2$	flow rate (production) of CO_2
PAL	pressure in the alveoli
$CLTH$	constant lung thorax compliance
FRC	functional residual capacity
$PVENT$	pressure in the ventilator
RET	resistance in the endotracheal tube
$RESIST$	total resistance to flow of Bain circuit arrangement
FAL	CO_2 fraction in the alveoli
FVT	CO_2 fraction in the ventilator
FCT	CO_2 fraction in the last segment of the circuit closest to the ventilator

INSPIRATION. Inspiration is achieved by a flow, $\dot{Q}VT$, generated by the ventilator; we assume the gas to be incompressible over the normal pressure range. Inspiration is represented by the following equations:

$$\dot{Q}BAIN = \dot{Q}VT \qquad (4)$$

$$VVT(T) = VVT(T - DT) - \dot{Q}VT \times DT - \dot{Q}SPL \times DT \quad \text{(volume of the ventilator)} \qquad (5)$$

$$\dot{Q}ET = \dot{Q}BAIN + FGF - \dot{Q}S \quad \text{(summation of flows around the elbow)} \qquad (6)$$

$$\dot{Q}TR = \dot{Q}ET \qquad (7)$$

$$VAL(T) = VAL(T - DT) + \dot{Q}TR \times DT + (1 - 1/RQ) \times \dot{Q}CO_2 \quad \text{(volume of the alveolar space)} \qquad (8)$$

The alveolar pressure, PAL, is determined on the basis of an assumed constant lung-thorax compliance, $CLTH$, and the functional residual capacity (FRC) value. The following equation gives this relation:

$$PAL(T) = [VAL(T) - FRC]/CLTH \quad (9)$$

The pressures in various parts of the system are calculated on the basis of Ohm's law.

EXPIRATION. When the pressure outside the ventilator bellows decreases, which also decreases the pressure in the system, the pressure difference between the alveolar space and the ventilator largely determines the expiratory flow. The positive flow direction is toward the patient, thus the signs change with expiration.

$$(PAL - PVENT) = -\dot{Q}ET \times RET - \dot{Q}BAIN \times RESIST \quad (10)$$

With equation 6 this can be solved for $-\dot{Q}ET$.

During expiration, if $\dot{Q}TR$, $\dot{Q}ET$, $\dot{Q}BAIN$ and $\dot{Q}VT$ are expressed as negative values, the equations 4 through 9 remain valid. However, $\dot{Q}VT$ becomes a dependent variable and can be determined by equations 10, 7, 6, and 4, respectively. For the flow through the spill valve (which operates with mechanical ventilation during the last part of expiration) and through the exhaust valve (which controls the outflow rate during filling of the bellows), quadratic pressure-flow relations are assumed as given in Table 1.

These equations produce the necessary information for the carrier flows and volumes to calculate the shift factors in the transport equations 1, 2, and 3 in the main text.

MATHEMATICAL FORMULATION OF THE ALVEOLAR SPACE. The alveolar space is regarded as one compartment, the volume of which varies with time. CO_2 delivery to this space is described as a constant inflow of CO_2 during the respiratory cycle. However, this inflow may be varied with time to represent decreased washout of CO_2, e.g., after a decrease in cardiac output, or increased CO_2 output, e.g., with fever.

If FAL represents the CO_2 fraction in the alveolar space, VAL the alveolar volume, and $\dot{Q}TR$ the tracheal flow rate, equation 11 determines the alveolar CO_2 volume during expiration.

$$VAL(T) \times FAL(T) = VAL(T - DT) \times FAL(T - DT) + \dot{Q}CO_2 \times DT - \dot{Q}TR(T - DT) \times DT \times FAL(T - DT) \quad (11)$$

The term to the left of the equal sign is the volume of CO_2 at time T. The first term to the right is the CO_2 volume in the alveoli at $(T - DT)$, the second term the volume of CO_2 that entered the alveolar space during the time DT, and the last term the volume of CO_2 that leaves the alveolar space during expiration.

Division of both sides of the equation by $VAL(T)$ yields the new alveolar fraction $FAL(T)$. During inspiration, the last term changes because the carrier flow rate becomes the inspiratory flow and the corresponding CO_2 fraction is the fraction in the trachea at that time instant. The airways connecting the alveolar space to the endotracheal tube are treated as a tube. The values for the alveolar volume and inspiratory and expiratory flow rates are calculated as shown above by means of equations 4 through 10.

MATHEMATICAL FORMULATION OF THE VENTILATOR. The description of the ventilator is similar to that of the alveolar space. The volume changes with time, while inflow and outflow occur through the same orifice. Additionally, a spill valve opens when its threshold pressure is exceeded during expiration.

The following equation yields the CO_2 volume in the ventilator during the patient's expiration (ventilator inflow):

$$VVT(T) \times FVT(T) = VVT(T - DT) \times FVT(T - DT) - \dot{Q}VT(T - DT) \times DT \times FCT(T - DT) - \dot{Q}SPL(T - DT) \times DT \times FVT(T - DT) \quad (12)$$

The first term on the right is the CO_2 volume in the ventilator at the preceding time instant; the second term the CO_2 volume that enters the ventilator based on the fraction of CO_2 in the tube between the Bain circuit and the ventilator ($\dot{Q}VT$ is negative during this phase); and the third term the volume of CO_2 that leaves the ventilator during spilling. If no spilling occurs, as in the first part of expiration, this term equals zero, since $\dot{Q}SPL$ will be zero.

During the patient's inspiration (ventilator outflow) the second term on the right becomes

$$-\dot{Q}VT(T - DT) \times DT \times FVT(T - DT) \quad (13)$$

because gas leaves the ventilator with the CO_2 fraction of the ventilator, FVT. The third term in equation 12 becomes equal to zero.

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